Section 4.4

Definitions of Trigonometric Functions of Any Angle

Let θ be an angle in standard position with (x, y) a point on the terminal side of θ and $r = \sqrt{x^2 + y^2} \neq 0$.

$$\sin\theta = \frac{y}{r}, \quad \cos\theta = \frac{x}{r}, \quad \tan\theta = \frac{y}{x}, x \neq 0, \quad \cot\theta = \frac{x}{y}, y \neq 0, \quad \sec\theta = \frac{r}{x}, x \neq 0, \quad \csc\theta = \frac{r}{y}, y \neq 0$$

Reference Angle

Let θ be an angle in standard position. Its reference angle is the acute angle θ' formed by the terminal side of the θ and the horizontal axis.

Quadrant II, $\theta' = \pi - \theta$ (radians), $\theta' = 180° - \theta$ (degrees).

Quadrant III, $\theta' = \theta - \pi$ (radians), $\theta' = \theta - 180°$ (degrees).

Quadrant IV, $\theta' = 2\pi - \theta$ (radians), $\theta' = 360° - \theta$ (degrees).

If $\theta < 0$ find its positive coterminal angle.

Evaluating Trigonometric Functions of Any Angle

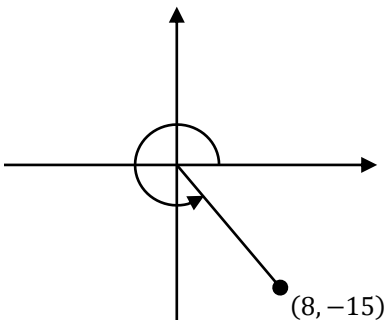
To find the value of a trigonometric function of any angle θ:

a) Determine the function value for the associated reference angle θ'.

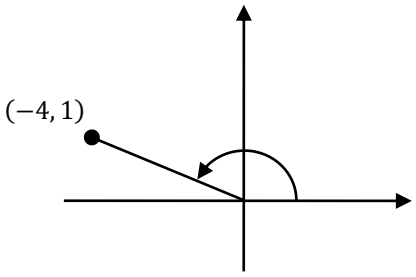
b) Depending on the quadrant in which θ lies, affix the appropriate sign to the function value.

Problem 1. Determine the exact value of the six trigonometric functions of the angle θ.

a)



b)



Problem 2. Find the values of the six trigonometric functions of θ.

a) $\cos\theta = -\frac{3}{5}$, and θ lies in quadrant II.

b) $\sec\theta = 3$, and $\tan\theta < 0$.

c) $\cot\theta$ is undefined, and $\frac{\pi}{2} \leq \theta \leq \frac{3\pi}{2}$.

Problem 3. Evaluate the sine, cosine, and tangent of the angle without using a calculator. (Find the reference angle).

a) $330°$

b) $-495°$

c) $\frac{5\pi}{3}$

d) $-\frac{11\pi}{6}$

Problem 4. Find the indicated trigonometric function value in the specified quadrant.

a) $\cos\theta = -\frac{4}{5}$, Quadrant II. Find $\sin\theta$.

b) $\tan\theta = -2$, Quadrant IV. Find $\cos\theta$.

c) $\sec\theta = -3$, Quadrant III, find $\cot\theta$.

Homework: Read section 4.4, do #9, 15, 21, 27, 35, 39, 49, 65, 71, 93, 97